Techniques for analyzing lens manufacturing data with optical design applications

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ABSTRACT

Optical designers assume a mathematically derived statistical distribution of the relevant design parameters for their Monte Carlo tolerancing simulations. However, there may be significant differences between the assumed distributions and the likely outcomes from manufacturing. Of particular interest for this study are the data analysis techniques and how they may be applied to optical and mechanical tolerance decisions. The effect of geometric factors and mechanical glass properties on lens manufacturability will be also be presented. Although the present work concerns lens grinding and polishing, some of the concepts and analysis techniques could also be applied to other processes such molding and single-point diamond turning.

Keywords: Optical design, statistics, optomechanical, manufacturing, distribution, nonparametric, lens tolerancing

1. INTRODUCTION

Lens design software generate optical tolerances by modeling a perturbed optical system to capture the effect of simultaneous changes in lens radius, center thickness, and other critical parameters in amounts matching the lens manufacturer’s stated capabilities. One step in the tolerancing process involves assumptions concerning the statistical distribution of the relevant parameters for a Monte Carlo simulation. In some cases, the presumed distributions do not resemble the actual lens manufacturing distributions. For example, optical designers assume a center thickness tolerance that is symmetric about the nominal value, but lens manufacturing brings parts to tolerance, not nominal values. The best option would use actual manufacturing data to build a statistical distribution for the Monte Carlo simulation. Understanding true manufacturing distributions will have other beneficial consequences. Insight into the fabrication process yields insight into the shop statistics; conversely, analyzing these statistics and distributions gives insight into the fabrication process. The desired results are a more cost-effective lens design and an increased probability that the fielded optical system will perform as expected.

The subject of optical tolerancing is actually a niche within the much larger tolerancing body of knowledge. Hong1 identified seven categories of tolerance research. The three categories most relevant to this discussion are (1) tolerance analysis (how to estimate assembly-level results given variability of the components), (2) tolerance synthesis (how to allocate the parts of an error budget), and (3) tolerance transfer (how to turn tolerancing decisions into a manufacturing plan). We will be using many concepts from the larger tolerancing body-of-knowledge in subsequent discussions. Our strategy is to base an analysis on actual manufacturing data, fabrication experience, and phenomenology; we propose that such an analysis can then better inform later strategic decisions related to tolerance allocation and transfer.

Measured lens data do not fit neatly into any mathematically derived format (Figure 1). Often, measured distributions are bimodal, indicating more than one process is at work, and distributions generally do not drop to zero at the edges. Taken together, these two observations suggest some of the lenses are polished a second time to pass inspection. The factors

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† As a single example, consider that Code V offers the designer a relatively conservative default choice of uniform distribution for the TOLMONTE analysis, (i.e., Monte Carlo perturbations of the system to derive tolerances), although other choices, such as a truncated Gaussian distribution, are available. However, all of the readily available options use distributions symmetric about the nominal value. The TOR option (i.e., wavefront differential tolerancing) also permits and END option that only allows for worst-case values of ±T.
that influence the shape of the distribution include shape (convex/concave/plano), mechanical glass properties, the precision grade (i.e., tolerance class), scratch-dig specifications, the R-number (radius-to-diameter ratio), the aspect-ratio (diameter-to-center thickness ratio), and the size of the batch (how many lenses being produced in the run). Manufacturing in this case can only remove material, so radii are ground and polished starting with larger (i.e., flatter) than nominal for a convex surface and smaller than nominal for a concave surface. An understanding of the grinding process is helpful in explaining the statistical differences between concave and convex surfaces.

Anticipating the possibility of rework, lens manufacturers typically leave center thicknesses larger than the nominal value. This approach is sometimes called the material-safe strategy. However, what makes sense from a manufacturing perspective may not from a lens design perspective: the fact that the mean value of center thickness is offset from the intended nominal value may create systematic deviations in the final assembly from the intended system metrics, such as wavefront error. This may further be exacerbated because the optomechanical mounting generally assumes that the nominal value is also the expected value. The end result is a systematic error in the distance-to-next value between optical surfaces. A mean value for some measured quantity that is different from the intended nominal value is often called a mean-shift.

We note that, in contrast to lens makers, machine shops generally do not use the material-safe strategy and generally aim for the center of the tolerance band as a strategy for maximizing yield. There are certain exceptions to this rule. Warpage that is due to the release of residual stresses can have a noticeable effect on a tricky form tolerance. For example, if a part has a difficult flatness specification, the machinist often would leave a little extra metal on that surface and take the final skim cut at the very end, if it is needed. But this type of material-safe strategy would most probably not apply to a typical lens barrel.

The mechanical design concept of “maximum material condition” (MMC) supposes that, at MMC, mechanical parts have the greatest probability of interference. However, this is quite different from the notion of material safety. Nevertheless, the idea that a mean-shifted probability density function (PDF) would affect the global assembly tolerances is well known and is usually referred to as “shifts and drifts” in the mechanical design literature. The machined features may systematically shift away from the nominal value due to setup errors, tool wear, or ambient temperature change. A common method for addressing this problem, called “benderizing,” defines the assembly error ($\epsilon_{assy}$) as

$$\epsilon_{assy} = C_f \sqrt{\sum_{i=1}^{n} \epsilon_i^2},$$

(1)

Where $\epsilon_i$ is the component error and $C_f$ (sometimes called a correction factor) is usually taken to be 1. However, to account for shifts and drifts Bender advocated using $C_f = 1.5$ based on experience, not mathematical analysis. Benderizing was popular and was adopted by companies such as Martin Marietta in their design procedures. Bothe gave a statistical explanation for the 1.5 factor 40 years after it was first proposed from the point of view of six-sigma quality programs.

Techniques of this type are referred to in the literature as RSS methods (i.e., root sum of the squares). Greenwood explains that in certain cases, the 1.5 factor may result in an accumulated tolerance that is actually worse than the theoretical maximum worst-case tolerance. Note that Eq. (1) represents a one-dimensional tolerance stack analysis. Chase refined the analysis of Greenwood and gave an overview of how to extend the RSS concept to a two- and three-dimensional tolerance stack. Chase discourages the

Figure 1. Binned and smooth histograms from a typical lens batch with 22 ground and polished lenses. The horizontal axes of all binned histograms are normalized.
use of Monte Carlo analysis due to the large number (up to $4 \times 10^5$) of simulations that need to be run assuming a 6σ assembly tolerance requirement, although this is less of a concern with more powerful computers. There is some awareness of shifts and drifts in the optical design literature. Lamontagne\textsuperscript{10} was aware of the Chase study, but abandoned RSS in favor of Monte Carlo methods to model optical assembly errors while accounting for shifts and drifts and the three-dimensionality of the assembly. Cheng\textsuperscript{11} discusses the use of the VSA-3D program (now called Tecnomatix Variation Analysis) with respect to Monte Carlo tolerance analysis and synthesis of optomechanical assemblies. CATS (computer-aided tolerance software such as VSA-3D) requires a solid model as input. Lamontagne, in contrast, deliberately avoids CATS in order to establish the tolerance allocation before the solid model is generated.

RSS methods are very well known in the optical tolerancing literature, although $C_f$ is usually taken to be 1, even though mean-shifted distributions are common in optical fabrication. Willey\textsuperscript{12} gives a detailed study of the tolerance allocation process using RSS methods and a quantitative cost analysis. Optical engineers often use the very conservative uniform distribution assumption in their Monte Carlo analysis, and it is possible that this conservative approach in tolerance analysis makes up for the shifts and drifts in optical fabrication.

The subject of optical tolerancing has a rich history,\textsuperscript{13,14} but there are no available studies that connect the parameters of lens grinding and polishing PDF data to the tolerancing practices of optical engineers. Juergens\textsuperscript{15} states that the engineers at Raytheon routinely look at lens production data from their suppliers and feed the resulting PDF estimations back into their Code V Monte Carlo tolerancing analysis. Tienvieri\textsuperscript{16,17} discusses a lens fabrication database and how it is used at Corning. But these data are not available to the public, as design houses vigilantly guard their optical designs, and fabrication houses work to prevent competitors from learning about the capabilities of their manufacturing processes.

The use of a database is important because (as Tienvieri notes), paper-based or even spreadsheet-based quality control methods are time consuming to maintain and are error prone. For serious work, a database with a standardized data entry format is needed. Two strategies for using shop statistics in the design process are (1) vertical integration and (2) quality inspection. With vertical integration, the design company has complete control over the fabrication process. With quality inspection, the design company would send quality inspection teams to the fabricator to agree on standards of data collection and process control. In both cases, the relationship between the designer and fabricator is far more complex than a simple bid process. The long term goal of such an effort would be to predict statistical distributions based on past history, or possibly to drive a manufacturing process to a desired outcome using statistical process control.

The ability to record, anticipate, or control the lens distributions has a number of benefits. The conversation of how to best organize an error budget is ongoing within the optics community. Optical and optomechanical engineers need data in order to intelligently allocate individual tolerances within the larger error budget. There are many situations where the demand for 100% interchangeability can be relaxed slightly in the interest of lower assembly tolerances and better system performance. A cohort of optical systems may be re-optimized based on the mean-shifted as-built lens batch data. And some instrument makers\textsuperscript{17,18} have been successful using selective assembly techniques.

2. FACTORS THAT INFLUENCE MANUFACTURING OUTCOMES

The lens maker is interested in many factors while choosing a manufacturing process that maximizes yield and ultimately profit, as shown in Figure 2. As we will see, a chosen process will leave signatures on statistical measures of manufacturing outcomes. Adding to the already formidable complexity of grinding and polishing is the fact that the phenomenology of loose abrasive grinding (sometimes called “three body” or “full aperture pitch polish”) and deterministic microgrinding (“two body” or “subaperture polishing”) are not governed by the same material factors.

We find that there is considerable spread in the way many of these factors affect statistical outcomes. There are threshold effects, a thin center thickness is usually not a problem until the aspect-ratio (center thickness to diameter) is greater than 10, for example. A difficult material will tend to have higher standard deviations, but there is significant noise in this assertion. The interaction effects between the factors are also important. In the same way that too many stresses on a patient would yield a poor medical outcome, too many stresses on a lens design would result in low manufacturing yield and a higher density on the boundaries of the manufacturing PDFs. In fact, many of the statistical insights presented here come from the bio-statistics world. Some of the factors that affect manufacturing outcomes are listed below.
**Tolerance class:** As we will see, the tolerance class has the most obvious effect on statistical outcomes. There are a limited number of fabrication processes available for any given set of material and geometric factors, and each of these leaves a signature in the statistical outcomes.

**Batch size:** We initially hypothesized that batch size would have a strong effect on manufacturing outcomes, due to different processes used, but we have found insufficient evidence to support this.

**R-number:** The ratio of optical radius to edge diameter (R/d) is an important parameter. The R-number extends from 0.5 (a hemisphere) to infinity (a flat surface). Multiple lenses can be blocked together (ground simultaneously) if the R-number is greater than 1.1. Lenses with a very small (less than 1) or a very large R-number (larger than 10) require special attention. In the correlation analysis we use (d/R) for convenience to avoid infinite values.

**Aspect-ratio:** The ratio of center thickness to edge diameter (d/CT) affects structural stability of the lens. An aspect-ratio of 6 to 12 is recommended. The act of polishing imparts a compressive residual stress to the glass; this is known as the “Twyman effect.” Thin lenses (i.e., aspect-ratio greater than 12) are more prone to movement after deblocking. Nearly planar surfaces have residual stress over the full clear aperture, whereas highly curved surfaces may only have residual stress at the outer rim. Willey\textsuperscript{12} links the aspect-ratio to figure irregularity and thermal effects rather than radius and residual stress effects.

**Cosmetic defects:** The scratch-dig ratio is an important driver when selecting a manufacturing process. The difficulty of meeting scratch-dig standards increases exponentially with the stringency of the specification. Process selection is driven by the ability to repair and recover from cosmetic imperfections found in-process, and stringent tolerances demand iterative, individual manufacturing approaches in anticipation of the high likelihood of cosmetic rework. Willey\textsuperscript{12} parametrizes the scratch dig ratio as (10/S + 5/D). This is what we use in the correlation analysis.

**Material factors:** We often think that soft glass (Knoop hardness, H\textsubscript{k} <400) is more difficult to polish, but glass is a deep and complex subject. Softer glass does require special attention, but there are many countermeasures that can be taken, such as polished bevels, rigorous cleanliness, mild solvents, and protective handling. A glass is truly difficult when prescriptive methods fail (if this, do that), resulting in unpredictable manufacturing outcomes, such that identical lenses made by identical processes can have divergent results. For example, some glasses fracture unpredictably upon deblocking.

There are a variety of mechanical sensitivities (related to fracture toughness and “Twyman effect”), thermal sensitivities (expansion/conductivity ratio), and chemical sensitivities (staining, solubility) that could require special procedures. For example, certain glasses are soluble in water (CaF\textsubscript{2} is an example) and in some cases cannot be processed with water-based slurries. It is often thought that glass that strays too far from the lead-line (sometimes called the glass line) will have fabrication issues. This line defines the right-most limit of the index vs. Abbe-number glass map.

**Ductility-index (DI):** This index\textsuperscript{20–22} is determined by fracture toughness and Knoop hardness (K\textsubscript{f}/H\textsubscript{k})\textsuperscript{2} and is given in units of length, often nanometers. A high DI would indicate greater susceptibility to residual stress and fabrication difficulties. Most optical glasses have a DI in the 10 to 40 nm range; KZnF\textsubscript{2} is 73 nm, MgF\textsubscript{2} is 60 nm, and CaF\textsubscript{2} is 125 nm. Unfortunately, fracture toughness data are not usually given in glass catalogs making this index hard to use.
**Stain class:** Willey\(^{12}\) gives a correlation of glass fabrication cost as a function of the stain code.

**Hardness-ratio \((H/H_{\text{calc}})\):** Because the causes of fabrication problems are complex, it is useful to have a single number that predicts fabrication difficulties due to material factors. We find that a ratio of measured hardness to predicted hardness (based on a linear fit of elasticity modulus to Knoop hardness as shown in Figure 3) of less than unity can predict fabrication difficulties. Figure 3 illustrates this concept with data from the Schott and Ohara catalogs. This approach has two problems: we don’t have a phenomenological explanation for it, and there may be exceptions. For example, the Kurtz family of glasses (signified by the letters Kz as in KzFSN\(_2\)) is notoriously difficult, but they are close to the glass-line and have a hardness ratio close to 1. We will be looking for the hardness ratio effect in the data analysis.

![Figure 3. Knoop hardness versus modulus of elasticity map, redder text indicates increased fabrication difficulty.](https://www.spiedigitallibrary.org/conference-proceedings-of-spie)

**3. ANALYSIS METHODS**

We use three general methods to analyze lens manufacturing data: parametric, nonparametric, and correlation analysis. Parametric methods are useful in characterizing the aggregate of many batches for later use in optical design programs. Nonparametric methods are primarily useful in characterizing individual batch statistics. Correlation analysis is a way of teasing out the factors that have the largest effect on a given output variable. Any method needs to consider the facts that the data are, by nature, bounded on some interval and may be bimodal.
3.1 Parametric methods

In a parametric approach the analyst selects a distribution, such as the Gaussian or Pearson, and then estimates the parameters for the distribution. Statistical distributions that are zero outside of a boundary are said to have compact support. There are special distributions for bounded datasets, such as the Beta distribution, but this distribution does not appear to be applicable in this study.

Operations may be applied to any distribution. Truncation is an operation that restricts the support of the distribution \( f(x) \) to the interval \((x_{\text{min}}, x_{\text{max}})\). The new truncated version of \( f(x) \) is given by \( f(x) / (F(x_{\text{max}}) - F(x_{\text{min}})) \), where \( F(x) \) is the cumulative distribution function of \( f(x) \). A mixture operation is a summation of two or more distributions such that the integral of the resulting PDF sums to 1. Bimodal data may be modeled as a mixture distribution. An important bimodal dataset (for testing new algorithms) is the wait-times between eruptions of the Old Faithful geyser. Although individual batches are often bimodal, aggregate statistics are generally unimodal.

Figure 4 is designed to help the reader understand how the truncation operation affects the meaning of the standard deviation. This truncation spans the interval \((-1, 1)\) for this example. A standard deviation above 0.577 indicates a multimodal distribution. Consider a mixture distribution with two modes centered on 1 and \(-1\) (the purple line in Figure 4). As the modes get sharper, the standard deviation for the mixture approaches 1. Therefore, the standard deviation of any normalized truncated distribution on \((-1, 1)\) cannot exceed 1. In the limit as the two modes become delta functions, the mixture becomes what Koch calls the end-point distribution; this is what Code V models with the TOR option.

Using the data presented here will ultimately involve fitting it to some distribution that can be used in an optics program as part of the Monte Carlo tolerancing process. After a suitable parametric distribution is found, one would estimate the optimum value of those parameters by maximizing a log-likelihood function. This is, in fact, a global optimization that may have local and global maxima; caution should be exercised in drawing quick conclusions.

3.2 Nonparametric methods

Nonparametric methods avoid assumptions regarding the shape of the PDF that best describes the data in question. But as we shall see, a few assumptions inevitably creep in. Binned histograms and nonparametric methods such as the kernel density estimate (KDE) are often used in exploratory data analysis. KDE methods have certain important advantages over the more commonly used binned histograms in that they yield a smooth probability distribution function. Because of this,
multiple datasets can be plotted concurrently. The ability to quickly compare multiple datasets is extremely important in exploratory analysis of the type we have done for this study. Resulting nonparametric PDFs can be employed just like parametric PDFs to calculate probabilities or to generate random variates. The kernel density estimate is calculated as

\[ f(x) = \frac{1}{nh} \sum_{i=1}^{n} K \left( \frac{x-x_i}{h} \right) \tag{2} \]

where \( f(x) \) is the PDF estimate, \( K \) is the kernel function, \( x_i \) is abscissa value of the \( i^{th} \) data point, \( n \) is the number of data points, and \( h \) is a smoothing parameter that is usually referred to as the bandwidth. The popular Gaussian kernel was used for this analysis,

\[ K(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} \tag{3} \]

The Silverman rule\(^{23}\) is often used to calculate the optimal bandwidth \( h \),

\[ h_{\text{optimal}} = \hat{\sigma} \cdot \left( \frac{4}{3n} \right)^{1/5} \approx 1.06 \cdot \hat{\sigma} \cdot n^{-1/5}, \tag{4} \]

where \( \hat{\sigma} \) is the calculated standard deviation of the dataset and \( n \) is the number of data points. There are other important methods for estimating the optimal bandwidth, for example the Sheather-Jones method.\(^{25}\) In a sense, the KDE algorithm behaves like a low-pass filter that attenuates features smaller than the bandwidth \( h \). If \( h \) is far below the Silverman optimum, the resulting PDF will have increased roughness and reflects the randomness within the data rather than its intrinsic structure. For a given bandwidth \( h \), more data points will reduce the roughness of the estimated PDF. An interesting consequence of the Silverman rule is that reducing \( h \) to resolve small features in the data would require a very large increase in the number of data points. Roughness is a statistical parameter that is calculated as

\[ R(f) = \int \left( \frac{\partial^2 f(x)}{\partial x^2} \right)^2 \, dx. \tag{5} \]

What are the hidden assumptions in the KDE method and the Silverman rule? The KDE method assumes that the dataset is unbounded but makes no assumptions about the shape of the distribution (i.e., uni-modal or multi-modal). The Silverman rule assumes that the data are close to a Gaussian shape (therefore unbounded and uni-modal). As an interesting note, Bernard Silverman was captivated by the bimodal Old Faithful data and was interested in the problem of bounded data as well.\(^{23}\) However, our problem involves bounded distributions that do not drop to zero at the edges and may be bimodal. But all is not lost; there is a body of knowledge to guide us through these difficulties.\(^{26}\)

Figure 5 shows PDF approximations generated by the KDE method (also called smooth histograms) of a uniform distribution with 5000 random data points over the interval \((-1, 1)\). These plots were created in Mathematica using the \textit{Histogram} and \textit{SmoothHistogram} functions, and the \textit{SmoothKernelDistribution} function to generate the PDF estimates. The truncation in the middle plot was done using the \textit{TruncationDistribution} function. The bounded PDF plot (Figure 5c) was generated using the reflection method as discussed by Silverman.\(^{23}\) The region in Figure 5a outside the \((-1, 1)\) interval is called spillover, and it constitutes about 10% of the overall PDF. The truncation algorithm was used in Figure 5b to cut away the spillover and proportionally increase the PDF inside the \((-1, 1)\) interval. The underestimated regions to the right and left in Figure 5b demonstrate the “edge bias” effect. This effect is mostly nullified by the reflection method as shown in 5c.

With the reflection method, Eq. (2) is modified as

\[
\begin{align*}
    f(x) &= \frac{1}{nh} \sum_{i=1}^{n} \left( K \left( \frac{x-x_i}{h} \right) + K \left( \frac{x-(2x_{\text{min}}-x_i)}{h} \right) + K \left( \frac{x-(2x_{\text{max}}-x_i)}{h} \right) \right) & \text{for } x_{\text{min}} \leq x \leq x_{\text{max}} \\
    f(x) &= 0 & \text{for } x_{\text{min}} > x > x_{\text{max}},
\end{align*}
\tag{6}
\]

\( x_{\text{min}} \) and \( x_{\text{max}} \) are the truncation points in the interval \((-1, 1)\).
where $x_{\text{min}}$ and $x_{\text{max}}$ are the right and left boundaries. In general, the sum of Eq. (6) will not result in a PDF that integrates to 1. However, Eq. (6) is only defined within the boundaries and zero everywhere else. Using this definition, the PDF will integrate to 1. The reflection method does have a flaw, however, as it forces the slope of the PDF to be zero at the boundaries. Consequently, the reflection method loses accuracy for data that are highly sloped at the boundaries. Many methods, called “boundary correction methods,” have been developed to correct this problem. Albers\textsuperscript{27} reviews some of these strategies and also provides the R-code used in evaluating them. We note that R is an open-source computer language designed for statistical computing and graphics, and is popular\textsuperscript{27} in part because it can be integrated with many database programs. Although we have used Mathematica in this study, more extensive work may involve studying R.

We spent time investigating many boundary correction methods. These methods are very complicated and should be avoided unless the analyst is prepared to spend a great deal of time. Generally, the PDFs we examined are not highly sloped at the boundaries; although Eq. (6) isn’t perfect, but it is good enough with one important exception. Irregularity and wedge are two optical errors that are given a maximum acceptable value; but these errors cannot actually be equal to zero. For convenience we assume that the PDF value is zero at the origin and that the PDF decreases as it approaches zero. For these the analyst may use

$$f(x) = \begin{cases} \frac{1}{nh} \sum_{i=1}^{n} \left( K\left(\frac{h}{x-x_i} - K\left(\frac{h}{x-(x_{\text{min}}-x_i)}\right) + K\left(\frac{h}{x-(x_{\text{max}}-x_i)}\right) \right) \right) & \text{for } x_{\text{min}} \leq x \leq x_{\text{max}} \\ 0 & \text{for } x_{\text{min}} > x > x_{\text{max}} \end{cases}$$

Eq. 7 forces the left PDF boundary to be equal to zero (using anti-reflection\textsuperscript{23}) without also forcing the slope to be zero. In many cases there will be a substantial slope at the left boundary for wedge and irregularity data. Unfortunately Eq. 7 will not result in a PDF that integrates to 1. Consequently, the analyst will need to integrate Eq. 7 for a correction factor that forces $f(x)$ to behave like a proper PDF.

A analyst that attempts to program Eqs. 6 and 7 will quickly discover that these methods become computationally expensive as the number of observations increases. An interpolation approximation of Eq. 6 or Eq. 7 can easily result in a 100-fold decrease in computation time. The distance between interpolation points should be less than $h/10$.

It is interesting that much of the research concerning bounded and mixture distributions\textsuperscript{28,29} focuses on the multivariate case. We have been concerned with the univariate data. Whereas the realities of boundedness and bimodality are merely annoying in the univariate case, they are positively vexing in the multivariate case. Suppose one wanted to understand the coupling between radius and irregularity errors. It would be desirable to investigate the joint probability distribution with respect to radius and irregularity. As the number of independent variables increases, statisticians start to talk about the “curse of dimensionality.” Bounded and bimodal datasets present a particularly interesting version of this curse.

How do we select a reasonable bandwidth given that our data violate the assumptions underlying the Silverman rule? In our 2014 study,\textsuperscript{30} we standardized on an $h$-value equal to 1/8 of the interval (0.25 for radius and center thickness, 0.125 for wedge and irregularity). This choice will yield relatively smooth curves, but can have an unintended consequence: a nonparametric curve will not resolve critical features if a dataset has a standard deviation that is less than the selected $h$-value. For the present study, we use the Silverman rule. However, it is imperfect, and in some cases we will get curves that show some roughness, but highly peaked distributions will be resolved more accurately.

![Figure 5. (a) Standard KDE algorithm, (b) truncated KDE algorithm, and (c) bounded KDE algorithm are shown with binned histograms (ordinate dimension normalized to the PDF function) for comparison. The horizontal axis is the interval of support, the vertical axis is the density value.](https://www.spiedigitallibrary.org/conference-proceedings-of-spie)
3.3 Correlation analysis

The correlation coefficient $r$ quantifies the strength of the relationship between the two variables. The sign of $r$ indicates the direction of the relationship. If $r$ is positive, the two variables rise and fall together; if $r$ is negative, one rises when the other falls. The coefficient of determination ($r^2$) indicates the proportion of variation that can be attributed to that relationship. The significance is the probability that the correlation is due to chance. A lower significance counterintuitively indicates higher confidence in the result. A correlation with a significance that is lower than 0.05 is considered a strong result.

A wise colleague made the following comment on the data and analysis we explored as part of this inquiry: “We know three things: we have some data, we understand some of the causal relationships, and we have intuition based on decades of experience.” Data can be limited and noisy; intuition is sometimes wrong; and our causal understanding may be incomplete, but together these tell a story, and we find that the three-way conversation has been very useful in refining our understanding and defining directions for exploration.

The output variables that we will be focusing on are the batch mean and standard deviation for radius, irregularity, and center thickness. Correlation analysis does have a few assumptions (linearity and normality, for example), some of which may not be completely valid. Several of the variables are known to have nonlinear relationships but we assume they are sufficiently linear for present purposes. The variables may not be normal, but batch statistics are roughly normal, so this is a good assumption. We have a limited amount of data, so the significance of our findings is also important.

4. EXPLORATORY DATA ANALYSIS (EDA), CORRELATION ANALYSIS, AND DATA-FITTING

The point of an EDA is to look at the data without making assumptions. As you might imagine, “looking” implies that graphical methods are important. Typical graphical tools include histograms, scatterplots, pairwise comparison charts, correlation analysis, and KDE plots. In the beginning, the analyst would examine the data for errors and outliers. Later on the analyst would look for relationships between dependent and independent variables. The question we (as designers) are interested in is how do the lens parameters (geometric factors, material factors, and tolerance class) affect the manufacturing and assembly outcome? And more to the point, how do we use this information to make our assembled systems more manufacturable? One can think of the EDA as a sub-step in the tolerance analysis prior to tolerance allocation and transfer.

In order to compare lens data for a variety of conditions, center thickness and radius data were normalized from $±T$ to $±1$ for this study. The surface irregularity and wedge data were normalized to (0, 1). Normalization has some unexpected benefits. As illustrated in Figure 4, the normalized standard deviation helps the analyst identify the number of modes. This fact gives the analyst a powerful tool to make quick assessments. For example, a dataset spanning the interval $(-1, 1)$ that appears to be unimodal but has a standard deviation larger than 0.577 probably has data entry problems. High standard deviations ($>0.54$) of measured (and normalized) radius or center thickness data would seem to indicate serious manufacturing problems.

One would expect that as the absolute value of these tolerances gets tighter, the normalized standard deviation would increase compared to the interval (and in some cases this is true).

This study examined twenty three batches (623 lenses total), and our prior study examined seven lens batches (182 lenses total). The word “batch” in this context refers to an order of nominally identical lens elements. In this study, the word “aggregate” refers to the statistics of all of the batches or lens elements mixed together. The lens data were supplied by Optimax Systems, Inc. Most of the lenses were ground from flat cylindrical stock; one started as a molded blank. Batches ranged in size from 3 to 176 lenses; of the 46 surfaces produced, half were convex and four were plano, the rest were concave. The R-number ranged from 0.5 to 15 (not including plano surfaces). We will be examining how the various factors influence the outcome.

Consider ETD (edge thickness deviation), also referred to as wedge, TIR or total indicator runout. We looked at how the tolerance class affects manufacturing data. The high tolerance ($T = 0.009$ to $0.005$ mm) and lower tolerance ($T = 0.01$ to $0.005$ mm)
0.025 mm) batches appear to be neatly separated (Figure 6) in a scatterplot. If we look at nonparametric plots of each
tolerance class, we find that there are qualitative differences with the lower tolerance curves highly peaked to the left and
the stringent tolerance curves more evenly distributed (Figures 7a and 7b). Because there are a limited number of possible
manufacturing processes, increasing the tolerance class stretches the distributions rightward.

Figure 6. Mean vs. standard deviation scatterplot of wedge data, colored according to tolerance value (Optimax data).

Figure 7. Nonparametric batch (thin blue) and aggregate (thick magenta) plots of wedge with best fit to parametric curves (black).

Eq. 7 (using the left-hand anti-reflection) was very successful in modeling the nonparametric curves for irregularity and
wedge. Eq. 6 would have poor accuracy due to the steep left-hand slopes in these quantities. In many cases the batch
standard deviation was small compared to the interval, so the Silverman optimal \( h \) (Eq. 4) was very important in capturing
the highly peaked curves. A 1/8 interval rule failed badly in this case. We fit the aggregate data to a truncated Gamma distribution (shown in Figures 7c and 7d). We also found that a truncated Skew-Normal distribution (a normal distribution with an extra skew parameter) could also have been used. The fact that the parametric and nonparametric curves agree is extremely important because you can’t completely trust either of them individually.

The results demonstrate that the typical assumption of a uniform distribution with respect to wedge is overly conservative except possibly in the case of an ETD tolerance of 0.005 mm. Wedge and optomechanical lens centration both impact boresighting; therefore, there will be a trade-off between the two. Knowing the manufacturing distributions would allow considerable relaxation of the optical and optomechanical tolerances without detriment to performance.

We approached the analysis of irregularity (IRR) in a slightly different way, as we were interested in teasing out the effects of tolerance class as well as other material and geometric factors. One of the challenges we encountered with graphical EDA methods is that it is very difficult to visually assess the effects of variables when one of them has a large effect and one has a small effect. A correlation analysis (Table 1) will not give us a pretty picture, but the numbers tell a fascinating story. Here we correlate the batch mean and standard deviation to the independent variables.

<table>
<thead>
<tr>
<th>Irregularity</th>
<th>Batch size</th>
<th>d/R</th>
<th>Tolerance</th>
<th>H/H(_{\text{calc}})</th>
<th>d/CT</th>
<th>H(_k)</th>
<th>Scratch-Dig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batch mean, (r)</td>
<td>0.067</td>
<td>0.146</td>
<td>0.372</td>
<td>0.058</td>
<td>0.415</td>
<td>–0.048</td>
<td></td>
</tr>
<tr>
<td>Batch mean, (r^2)</td>
<td>0.004</td>
<td>0.021</td>
<td>0.138</td>
<td>0.003</td>
<td>0.172</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>Batch mean, sig</td>
<td>0.092</td>
<td>0.378</td>
<td>0.145</td>
<td>0.851</td>
<td>0.035</td>
<td>0.174</td>
<td></td>
</tr>
<tr>
<td>Batch Std. Dev., (r)</td>
<td>0.335</td>
<td>0.316</td>
<td>0.112</td>
<td>0.35</td>
<td>0.242</td>
<td>–0.046</td>
<td></td>
</tr>
<tr>
<td>Batch Std. Dev., (r^2)</td>
<td>0.112</td>
<td>0.1</td>
<td>0.013</td>
<td>0.123</td>
<td>0.059</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>Batch Std. Dev., sig</td>
<td>0.046</td>
<td>0.05</td>
<td>0.005</td>
<td>0.803</td>
<td>0.055</td>
<td>0.27</td>
<td></td>
</tr>
</tbody>
</table>

We see that the tolerance class is strongly correlated with the batch mean (no surprise) and the batch standard deviation with very high confidence (again, significance is low). The \(r^2\) coefficient of determination for batch mean indicates that 19% of the data variation is due to the tolerance value. We see that hardness is positively correlated to the batch mean but not the standard deviation. In other words, a harder glass would on average have a (counterintuitively) worse measured irregularity, but the data spread would not be affected. Also notice that hardness rather than hardness ratio has the stronger correlation (look at the significance numbers). The R-number has an interesting effect. As the surface curvature increases, the data spread gets larger with no effect on the average value.

Some of the results of the correlation analysis seem counterintuitive to the novice and a bit noisy to the data analyst. We would have thought softer glass more problematic. Consider the three worst-case nonparametric curves in Figure 8b. These have right-hand boundary density values between 0.9 and 2. As it turns out, they also have the highest hardness numbers (1100 and 700 Knoop hardness) and relatively low R-numbers (between 1.26 and 0.9); this fits well with our correlation analysis, but what does it mean? High hardness glass generally requires more force during grinding, and the force is directed downward (parallel to the optical axis). But if there also is a strong curvature, the edge is seeing a different force than the center, thereby leading to irregularity issues. There are special techniques that can be used to avoid this problem, but apparently they weren’t used with these lenses. The coupling between hardness and R-number in the irregularity data is a good example of the many interaction effects that make lens polishing so complicated.

Again, the parametric irregularity curves fit nicely to a truncated Gamma distribution. An interesting philosophical question came up while making these plots. One can think of tolerance allocation as being analogous to risk management. Suppose a designer starts using this esoteric Gamma distribution in Monte Carlo analysis with the most accurate estimate of the parameters from historical data. But is this the best idea? The solid black line in Figure 8d is a best fit with the Gamma shape (k) and scale (\(\theta\)) parameters equal to 4.33 and 0.124, respectively. Suppose we artificially inflated the shape parameter to 5.0 (the dotted black line). Now we have a parametric curve that is not particularly accurate, but slightly more conservative, thereby lowering overall risk during the tolerance analysis phase. Which is the better choice?

Moving on to analysis of the center thickness data (Table 2), we see that a stringent center thickness tolerance primarily impacts the data spread, but not the mean. For every lens, we expect some limiting tolerance that dominates fabrication difficulty. Since the give in the center thickness is frequently used to mitigate tight tolerances elsewhere, it may be that...
the center thickness mean is better correlated to the limiting tolerance, whatever it may be, than to the center thickness tolerance. We hypothesized that as the hardness ratio increases, glass processing may be more deterministic, therefore requiring less material removal and moving the mean to the right, but the data are not convincing. A prior analysis seemed to show a trend regarding hardness ratio and batch mean (with 0.09 significance), but a couple of extra datasets weakened the trend considerably. It is easy to see a result that is not there if you have an internal bias and the data are sparse.

![Figure 8. Nonparametric batch (thin blue) and aggregate (thick magenta) plots of irregularity with data fit to parametric distributions (black).](image)

Table 2. Effect of material and geometric factors on center thickness using correlation methods.

<table>
<thead>
<tr>
<th>Center thickness</th>
<th>Batch size</th>
<th>d/R</th>
<th>Tolerance</th>
<th>H/H_{calc}</th>
<th>d/CT</th>
<th>H_{k}</th>
<th>Scratch-Dig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batch mean, r</td>
<td>-0.08</td>
<td>-0.22</td>
<td>0.21</td>
<td>0.24</td>
<td>-0.06</td>
<td>0.13</td>
<td>0.31</td>
</tr>
<tr>
<td>Batch mean, r^2</td>
<td>0.0064</td>
<td>0.0484</td>
<td>0.0441</td>
<td>0.0576</td>
<td>0.0036</td>
<td>0.0169</td>
<td>0.0961</td>
</tr>
<tr>
<td>Batch mean, sig</td>
<td>0.61</td>
<td>0.31</td>
<td>0.33</td>
<td>0.27</td>
<td>0.85</td>
<td>0.5</td>
<td>0.28</td>
</tr>
<tr>
<td>Batch Std. Dev., r</td>
<td>0.15</td>
<td>0.41</td>
<td>-0.44</td>
<td>-0.14</td>
<td>-0.18</td>
<td>0.02</td>
<td>-0.25</td>
</tr>
<tr>
<td>Batch Std. Dev., r^2</td>
<td>0.0225</td>
<td>0.1681</td>
<td>0.1936</td>
<td>0.0196</td>
<td>0.0324</td>
<td>0.0004</td>
<td>0.0625</td>
</tr>
<tr>
<td>Batch Std. Dev., sig</td>
<td>0.96</td>
<td>0.5</td>
<td>0.04</td>
<td>0.39</td>
<td>1.0</td>
<td>0.45</td>
<td>0.78</td>
</tr>
</tbody>
</table>

We found that 4 out of the 23 lens batches had significant left-hand boundary densities, being 0.15, 0.21, 0.21, and 0.51. The worst offender was a lens with an R-number of 0.5, a center thickness tolerance of 0.02 mm (very stringent), and a difficult material as measured by the hardness ratio. It is not clear why the other three batches ended up with significant numbers of thin lenses. It may be possible to predict thin lenses by looking at combinations of worst-case parameters. Lens tolerancing generally assumes a nominal center thickness that is in the middle of the tolerance band, but this assumption is more likely to hold with lenses that are difficult to make.

Figure 9 shows both parametric and nonparametric center thickness curves. The parametric curve fit well with a truncated normal distribution using the parameters $N(0.73, 0.66)$. The black dotted curve using the values $N(0.73, 0.86)$ is an attempt (using a higher standard deviation parameter) to model a slightly more conservative parametric curve for our tolerance analysis.
Kaufman\textsuperscript{30} outlined a procedure for using an asymmetric tolerance zone for center thickness given that the mean value is usually not symmetric, under the premise that a better design has an expected value for center thickness that is closer to the nominal design value. Our hope is that a designer will be able to relax tolerances for center thickness and lens-to-lens spacing using historical data with this procedure. This may not be worth doing for a low-performance system, but it could be very useful for a high-performance system. A lens with a difficult geometry and material is often subject to rework, so a small increase in center thickness tolerance reduces risk to the fabricator, and both cost and risk to the designer.

The lens radius of curvature is usually the most critical optical variable. The radius deviation was given in millimeters (i.e., nominal radius \( \pm \Delta R \)). We converted between radius change and interferometric fringes with the formula

\[
\text{Number of fringes} = \frac{\varphi^2 \Delta R}{4 R^2 \lambda}, \tag{8}
\]

where \( \lambda \) is the measurement wavelength (taken to be 632.8 nm), \( R \) is the nominal radius, \( \Delta R \) is the tolerance, and \( \varphi \) is the aperture diameter. The radius tolerances ranged from 0.6 to 18 fringes.

Table 3. Correlation results for radius data.

<table>
<thead>
<tr>
<th>Mean-shift</th>
<th>Batch size</th>
<th>d/R</th>
<th>Tolerance</th>
<th>H/H\textsubscript{calc}</th>
<th>d/CT</th>
<th>H\textsubscript{k}</th>
<th>Scratch-Dig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batch mean, ( r )</td>
<td>0.234</td>
<td>0.055</td>
<td>0.025</td>
<td>(-0.458)</td>
<td>(-0.021)</td>
<td>(-0.309)</td>
<td>(-0.119)</td>
</tr>
<tr>
<td>Batch mean, ( r^2 )</td>
<td>0.055</td>
<td>0.003</td>
<td>0.001</td>
<td>0.21</td>
<td>0</td>
<td>0.095</td>
<td>0.014</td>
</tr>
<tr>
<td>Batch mean, sig</td>
<td>0.223</td>
<td>0.727</td>
<td>0.805</td>
<td>0.002</td>
<td>0.84</td>
<td>1.27</td>
<td>0.014</td>
</tr>
<tr>
<td>Batch Std. Dev., ( r )</td>
<td>0.008</td>
<td>0.08</td>
<td>(-0.078)</td>
<td>(-0.032)</td>
<td>0.127</td>
<td>(-0.062)</td>
<td>0.072</td>
</tr>
<tr>
<td>Batch Std. Dev., ( r^2 )</td>
<td>0.0</td>
<td>0.006</td>
<td>0.006</td>
<td>0.001</td>
<td>0.016</td>
<td>0.004</td>
<td>0.005</td>
</tr>
<tr>
<td>Batch Std. Dev., sig</td>
<td>0.889</td>
<td>0.419</td>
<td>0.835</td>
<td>0.846</td>
<td>0.576</td>
<td>0.523</td>
<td>0.873</td>
</tr>
</tbody>
</table>

In our correlation analysis (Table 3), we use the absolute value of the batch mean as the independent variable, since we are interested in understanding the amount of mean-shift rather than its sign. We see that the hardness ratio (rather than the hardness or tolerance class) is the dominant effect, accounting for about 20% of the total variation. We expected a lower hardness ratio to increase the probability of a mean shift, and that is what we found. The batch standard deviation, on the other hand, seems to be completely random. Experience teaches that a very low R-number (<1) can be problematic, but that very large R-numbers (>10) can also be problematic, making it difficult to detect a trend. We excluded planar surfaces from our analysis because, statistically, they vary substantially from curved surfaces.

Figure 10 shows the parametric and nonparametric distribution curves for radius data. The data mean was \(-0.06\) the standard deviation was 0.52. The data were fit to a truncated normal distribution, \( N(-0.16, 0.79) \). The radius data for our 2014 study was fit to \( N(0.18, 1.5) \). Figure 11 graphically shows the hardness ratio effect. The left plot shows radius data for easy glass as measured by the hardness ratio; the right plot shows radius data for difficult glass. The aggregate plot for difficult glass is very close to a uniform distribution, and the batch plots have much higher edge densities.
5. DISCUSSION

Our goal is to develop methods for understanding manufacturing outcomes and then using the result during the tolerancing phase of the design process. We used a scatterplot to categorize the effects of a primary independent variable. We also used correlation analysis to gauge the strength of many independent variables. We then used nonparametric methods to visualize the batch and aggregate manufacturing outcomes. Finally, we used parametric methods to characterize the outcome. The parametric approximation may be used by optical programs (with some customization) for tolerancing.

We employed exploratory data analysis to identify trends in the data without making too many assumptions. Correlation analysis proved to be an excellent guide to help us understand the most important effects. However, the data are relatively sparse and often noisy, so intuition based on experience and an understanding of the causal mechanisms provided crucial guidance, particularly in cases of expected interactions between variables. Intuition is also helpful in understanding data when a contributor reaches a trigger point, an extreme value, or has a nonlinear effect. The three-way conversation proved to be useful, for example, when seeking to understand why high hardness, stringent tolerances, and a low R-number would together cause irregularity issues. Conversely, even dispassionate engineers can become invested in a particular bias, and it can be difficult to admit that the numerical evidence is weak.

One of the things nonparametric KDE methods can do is estimate the density value at the boundaries. It is true that the method we used has some accuracy issues on the boundaries, but we are looking for a ranking of worst-case results to help in understanding patterns in the data.

In a recent design, we found that our lens centration tolerances were too tight and wedge tolerances too loose based on data such as shown in Figure 7 and discussions with our fabricator. As a result, we shifted the tolerance allocation and found that we could significantly loosen our centration tolerances while maintaining performance. By doing this, we
essentially shifted risk from the assembly operation to the manufacturing operation. The additional risk to the fabricator was small, given the more stringent tolerances were well within his process capability, whereas the risk reduction to the assembly step was substantial. We expect that we could have loosened assembly tolerances even more had we made use of expected distributions in the Monte Carlo tolerance analysis. A small increase in lens cost can be justified if the overall project cost is reduced.

The notions of tolerance allocation and risk management are deeply related. For example, how much trust should we put in an expected distribution based on historical data, and is there a real benefit to using them as opposed to the standard uniform distribution? Are there ways of tweaking the expected distribution to lower overall risk? In the case of wedge and irregularity it seems clear that shifting the expected distribution to the right is a way of lowering overall risk. In the case of center thickness and radius, it is not so clear. One idea (shown in Figure 9) is to artificially increase the standard deviation parameter so that the resulting distribution is a little closer to a uniform distribution. Of fundamental importance here is the observation that the expected distribution exists within a confidence envelope, so the batch distributions may vary more than one would expect from a typical Monte Carlo simulation.

It is very well known that lenses tend to come in thicker than nominal, but this is rarely quantified. It occasionally happens that lenses are also thin. We had hoped to isolate the factors that would cause thin lenses so we could better predict the outcome. Our intuition tells us that if there are too many stresses on the manufacturing process, the outcome become unpredictable and more material removal would be needed. Kaufman\textsuperscript{30} gave specific ideas on how to structure an asymmetrical center thickness tolerance zone so that the expected value is much closer to the nominal value.

In our 2014 study\textsuperscript{30}, we modeled the optical parameters using the truncated normal distribution almost exclusively. In this study we used the Gamma distribution to model wedge and irregularity. We found that the three-parameter Skew-Normal distribution could also have been used. The problem is that more parameters dramatically increase the difficulty in finding an optimal fit to data, because we are looking for a global maximum to the log-likelihood function and might get stuck in a local maximum with a less intensive search. This is one of the reasons why it is a good idea to have multiple, independent ways of looking at the data. We propose that any PDF modeling wedge and irregularity should have a zero density at the leftmost boundary, and should be decreasing as it approaches that boundary, as it is not physically possible to have zero error. By using this technique, it is easy to see how making the tolerance more stringent stretches the distribution to the right.

We found that correlation analysis is a wonderful tool for teasing out the impact of many independent variables and a good compliment to the graphical methods. We also found that it is very important to understand the sign of the correlation coefficient ($r$), its value, the coefficient of determination ($r^2$), and its significance. In the process of doing the correlation analysis, we were able to confirm the intuition that the hardness ratio is a better predictor of manufacturing issues for radius than the actual hardness number.

A simpler version of the hardness ratio is also possible: $H_r/E$ (Knoop hardness as given in the glass catalog to modulus of elasticity measured in GPa) on average is equal to 6.3. A smaller value for this ratio is equivalent to $H/H_{calc}$ less than unity. The correlation results in Table 3 are the same with the simpler version of the hardness ratio.
6. CONCLUSION

This study seeks to document the shop statistics that could help optical engineers in the tolerance phase of design. In the process, we worked with statistical methods and tools not widely known in the optics community.

Lens fabrication is the result of highly skilled labor, but glass is a rather truculent material, and modern optical designs can be very demanding. Batch statistics sometimes have a clumpy character. A careful inspection of the sample batch plotted in Figure 1a shows that the smaller mode has very little variance. Opticians actively learn about how a particular glass responds as they work the material. In many of the figures, we see that individual batch distributions occasionally have a character that seems very different from that of the aggregate distribution. Hence, batch statistics are a fascinating mixture of deterministic lens polishing and unexpected randomness.

Despite the small size of this study, certain basic trends can be noted. Mean center thickness is usually larger than the nominal value assuming a symmetric tolerance zone. Center thickness does not seem to be the most important optical parameter, but when errors occur systematically they can have a powerful effect. Center thickness affects the optomechanical distance-to-next parameter that governs lens spacing, and systematic center thickness errors lead to unpredicted focus and spherical aberration.

We can see the effects of process on statistical outcomes, particularly irregularity. With irregularity, we see the precision class determines the fabrication process, which, in turn, affects the statistical outcome. We suspected that glass hardness or softness is an important statistical parameter in understanding irregularity, and we were able to refine this understanding considerably.

We have shown uniform distribution is a reasonable approximation for radius outcomes. But we also see that radius outcomes are frequently mean-shifted. We found evidence to support our intuition that the hardness ratio is an important parameter, and we were able to correlate this parameter to the radius mean-shift. We do not have a phenomenological explanation for this result.

Individual correlations such as the one above may be disputed in future studies. The more important labor is a methodology for examining optical manufacturing data from the design point of view. We know that many companies have commissioned in-house studies of this type. Because the results of these studies are not publically available, the methodologies for reaching these results are also not publically discussed. A future direction would be to show the impact of manufacturing data analysis on an optical design.

The concepts of interchangeability and tolerances, along with the role of craftsmanship, have a fascinating intertwined history hundreds of years old. Eli Whitney popularized the concept of part interchangeability in the 1790s, for example. From the time of Galileo and Spinoza until today, craftsmanship will always have a place in optics. But with specialization, the designer and fabricator are often far removed from one another. Smaller design houses are at a particular disadvantage because their access to fabrication data is somewhat limited.

We hope that this study encourages more lens suppliers to share fabrication data and engineers to view inspection reports with keener attention. Which is less expensive—configuring for 100% interchangeability or lowering tolerances, which will sacrifice some interchangeability, and then re-optimizing based on batch statistics? In many cases, the optical engineers receive the inspection reports from the fabrication shop, but do not subject the data to a trend analysis. We also hope to encourage more optical engineers to think strategically about the design-fabrication-assembly process.

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REFERENCES


