Slope Error Tolerances for Optical Surfaces

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The increased use of modern machining techniques for the production of optical surfaces (for spherical surfaces as well as aspheres) is changing the types of surface form error that need to be addressed as part of the overall tolerancing of an optical system.

Three commonly used metrics for tolerancing aspheric surfaces are Peak-to-Valley (PV) surface error, RMS variance of a surface, and tolerances placed directly on the aspheric coefficients that define the aspheric surface. We examined the efficacy of these three methods using a Monte Carlo simulation of 100 randomly perturbed versions of the Petzval lens shown in Figure 1.

This lens, designed specifically for this investigation, is a modification of a classical Petzval portrait lens. We used three aspheric surfaces (indicated in Fig. 1), and corrected all aberrations (including distortion) to very low values across a 7 degree full field. The lens operates at F/2.8.

As a simple test of the suitability of PV error, RMS error, and tolerances on aspheric coefficients, we randomly varied the 4 polynomial coefficients that control the $r^4$, $r^6$, $r^8$, and $r^{10}$ aspheric departures of the stop surface. In each case, we varied all four coefficients independently of each other. The maximum departure of each coefficient from its design value was constrained to a value corresponding to one-half wavelength of wavefront aberration at the edge of the aperture. (Thus, the worst-case for all four surfaces together would be 2 wavelengths of aberration.) In each case we calculated the PV and RMS departures of the wavefront; these are related to the PV and RMS of the surface, over the area of the beam. We also calculated in each case the Strehl ratio, defined as the ratio of the peak intensity of the image of a point source to the corresponding intensity for an aberration-free image. The Strehl ratio is also equal to the ratio of the volume under the MTF to the corresponding volume for an aberration-free system, and this makes it a useful metric of image quality. (We calculated the Strehl ratio by direct simulation of the Point Spread Function, rather than by a formula that depend on the RMS wavefront variance.) The results for PV wavefront departure and RMS wavefront departure are shown in Figures 2 and 3 respectively. In Fig. 2, a reasonable correlation between PV wavefront departure and Strehl is evident, and the familiar "Rayleigh quarter wave" criterion is indicated by the dashed blue lines. Note though, that there is sufficient scatter in the data that a PV value of 0.25 $\lambda$ could correspond to any Strehl value between 0.7 and 0.82. It can be seen that the quarter wave criterion holds well for some wavefront shapes, but less well for...
others. Figure 3 demonstrates the much better correlation between Strehl and RMS wavefront, for all wavefronts examined.

![Strehl vs. PV, 100 cases](image1)

![Strehl vs. RMS, 100 Cases](image2)

Fig. 2. Strehl Ratio vs. PV Wavefront  
Fig. 3. Strehl Ratio vs. RMS Wavefront

The solid line in the drawing indicates the theoretical dependence, Strehl = exp(-2πRMS)². The dotted lines in this case indicate the Marechal criterion, that an RMS of 0.071 λ or better corresponds to a Strehl Ratio of 82% or better.

In Fig. 4, we plot the relationship (or lack thereof) between the Strehl ratio and the fourth-order aspheric coefficient of the asphere. In the figure, points near the vertical axis with Strehl values near zero indicate that it is possible for very small departures of the aspheric coefficients to completely destroy the image quality. On the other hand, there are points on the figure, far from the vertical axis, with Strehl values near unity. These points demonstrate the degree of balancing that can occur among the various polynomial aspheric coefficients. One coefficient may depart significantly from its design value, yet this error can be almost entirely compensated for by corresponding deviations of the other aspheric coefficients from their design values. As we did not optimize the other aberration coefficients to correct for the error in the fourth-order coefficient, the balancing that is evident in the figure occurred by random chance. In practice, this balancing occurs intentionally, as the fabricator attempts to prevent a small error at an intermediate zone from propagating, with ever greater magnitude, to the edge of the part.

![Strehl vs. Aspheric Coefficient](image3)

Fig. 4. Strehl Ratio versus 4th-order aspheric coefficient
As a result, one typically finds that the "errors" in the polynomial coefficients alternate in sign, with the result that the surface exhibits some waviness, but generally adheres reasonably well to the desired form. The fact that the polynomial coefficients can be used to balance each other is a direct consequence of their not being mathematically orthogonal. When substantial balancing among parameters is possible, it is not possible to treat a departure of any individual coefficient from its desired value as an "error". For these reasons, it is not recommended that tolerances be assigned to polynomial coefficients defining an aspheric.

Although Fig. 3 indicates very good correlation between Strehl Ratio and RMS wavefront, Strehl Ratio alone is not always an adequate measure of system performance, particularly when ripple is present on the surface. Figures 5 and 6, for example, show the effect of concentric, sinusoidal surface ripple on the MTF. Figure 5 shows the case of four ripples across the lens aperture, and Fig. 6 shows the case of 32 ripples per aperture. In both cases, we adjusted the amplitude of the ripple so that a Strehl ratio of 82% was achieved. As the number of ripples across the aperture increases, the MTF approaches the theoretical limit for diffusely scattered light, which is that the MTF at all frequencies is reduced from the diffraction-limited value by a factor equal to the Strehl ratio. Note that in Fig. 6, the MTF drops precipitously to 82% of the diffraction limited value at very low spatial frequencies, whereas in Fig. 5, the MTF falls off more slowly.

To show the visual impact of these MTF curves, we simulated corresponding images of bar charts; these appear in Figs. 7 and 8. Comparison of the images shows that although the resolution is not much affected, the visual appearance of the image is greatly impacted by the loss of contrast at the very lowest frequencies.
We investigated the possible use of Peak Surface Slope error (PSS) as a tolerance metric to limit the impact of surface ripple on the low-frequency image contrast. To do this, we added concentric, sinusoidal surface ripple, at various spatial frequencies, to the optical surface at the stop. In each case, we chose the depth of the ripple to give a predetermined PSS. (The allowed depth increases as the frequency decreases, and at low spatial frequencies, the Strehl ratio falls well below 82%.) Figures 9 and 10 plot the MTF functions for several ripple frequencies, for PSS values of 75 microradians and 50 microradians, respectively.

We concluded that Peak Surface Slope error (the peak difference from the design surface) is a useful metric for controlling low frequency contrast loss caused by both surface ripple and errors in the polynomial aspheric terms. We recommend against placing tolerances directly on polynomial aspheric coefficients.